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LETTER TO THE EDITOR

Comparisons of phase times with tunnelling times based on absorption probabilities

Y Z Huang and C M Wang
Institute of Semiconductors, Chinese Academy of Sciences, PO Box 912, Beijing 100083, People's Republic of China

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Abstract. The times spent by an electron in a scattering event or tunnelling through a potential barrier are investigated using a method based on the absorption probabilities. The reflection and transmission times derived from this method are equal to the local Larmor times if the transmission and reflection probability amplitudes are complex analytic functions of the complex potential. The numerical results show that they coincide with the phase times except as the incident electron energy approaches zero or when the transmission probability is too small. If the imaginary potential covers the whole space the tunnelling times are again equal to the phase times. The results show that the tunnelling times based on absorption probabilities are the best of the various candidates.

The old question of how long it takes for an electron to tunnel through a barrier has received considerable attention, and several solutions have been proposed. Reviewing the proposed tunnelling times, Hauge and Støvneng [1] concluded that none of them provide a satisfactory answer to the basic question, and the dwell time [2, 3] and the phase time [4, 5] are the only well-established times in this context. The dwell time τ_D, defined as the average number of electrons in a region divided by the particle flux of the incident beam, offers an exact value for the time spent in a region of space, averaged over all incoming particles. The phase times τ_T^φ and τ_R^φ are asymptotic results for completed scattering events and include self-interference delays as well as the time spent in a barrier [1]. Numerical simulation shows that the traversal time of a wavepacket through a barrier agrees with the phase time rather well [6].

Recently, adding an imaginary potential in the tunnelling barrier to probe the reflection and traversal times was considered. The analysis showed that the absorption probability caused by adding an imaginary potential in the barrier does not distinguish between transmission and reflection, and consequently can only yield the dwell time [7]. In this letter, we show that the reflection time τ_R^a and transmission time τ_T^a based on the recombination process are equal to the tunnelling times derived from the Larmor precession [8–10] if the transmission and reflection probability amplitudes are complex analytic functions of the complex potential. Moreover, we have $\tau_R^a = \tau_R^\varphi$ and $\tau_T^a = \tau_T^\varphi$ when the whole space including the regions outside the barrier is covered by the imaginary potential.

For an incident beam of particles approaching an arbitrary barrier $V(z)$ $(0 \leqslant z \leqslant d)$ from the left, the stationary-state scattering wavefunction exterior to the barrier region is given by $\psi(z) = \exp(\mathrm{i}kz) + R\exp(-\mathrm{i}kz)$ for $z < 0$ and $T\exp(\mathrm{i}k(z-d))$ for $z > d$. If the potential energy is taken to be zero outside the barrier region, we should have total probability conservation $|R|^2 + |T|^2 = 1$. By assuming that the potential has a small imaginary part ΔV_{I}, the decay of the probability of the electron in the barrier can be described [11]. The decay time of the electron density modelled by the imaginary potential is

$$\tau = -\hbar/2\Delta V_{\mathrm{I}}. \tag{1}$$

With the imaginary potential, the total probability of reflection and transmission will depend on the length of time for which particles stay in the region. A dwell time $\tau_{\mathrm{D}}^{\mathrm{a}}$, which is the mean time spent by an incident particle of energy E in the region $0 < z < d$, can be defined by

$$|R|^2 + |T|^2 = \exp(-\tau_{\mathrm{D}}^{\mathrm{a}}/\tau). \tag{2}$$

The reflection and transmission amplitudes R and T are only related to the size of the barrier and the wavevectors in each region, the wavevector is

$$k = [2m(E-V)]^{1/2}/\hbar \tag{3}$$

where m is the electron effective mass and $V = V(z) + \mathrm{i}\,\Delta V_{\mathrm{I}}\,\theta(z)\theta(z-d)$ is the complex potential energy ($\theta(z)$ is the unit Heaviside function). In the limit of a small imaginary potential, the dwell time derived from (1) and (2) is

$$\tau_{\mathrm{D}}^{\mathrm{a}} = (\hbar/2)\,\partial(|T|^2 + |R|^2)/\partial\Delta V_{\mathrm{I}}|_{\Delta V_{\mathrm{I}}=0} \tag{4}$$

with $|R|^2 + |T|^2 = 1$ at $\Delta V_{\mathrm{I}} = 0$. If the interference between the reflection and transmission waves can be ignored, we can define a transmission time $\tau_{\mathrm{T}}^{\mathrm{a}}$, the mean time if the particles are finally transmitted, by

$$|T(\Delta V_{\mathrm{I}})|^2/|T(\Delta V_{\mathrm{I}} = 0)|^2 = \exp(-\tau_{\mathrm{T}}^{\mathrm{a}}/\tau). \tag{5}$$

In the limit of a small imaginary potential, the transmission time is

$$\tau_{\mathrm{T}}^{\mathrm{a}} = (\hbar/2)\,\partial\ln|T|^2/\partial\,\Delta V_{\mathrm{I}}|_{\Delta V_{\mathrm{I}}=0}. \tag{6}$$

In the same way, the reflection time $\tau_{\mathrm{R}}^{\mathrm{a}}$, the mean time if particles are finally reflected, is

$$\tau_{\mathrm{R}}^{\mathrm{a}} = (\hbar/2)\,\partial\ln|R|^2/\partial\Delta V_{\mathrm{I}}|_{\Delta V_{\mathrm{I}}=0}. \tag{7}$$

It can be easily found that the tunnelling times satisfy the probabilistic rule $\tau_{\mathrm{D}}^{\mathrm{a}} = |T|^2\tau_{\mathrm{T}}^{\mathrm{a}} + |R|^2\tau_{\mathrm{R}}^{\mathrm{a}}$, which was used to examine different definitions [1, 10]. Golub *et al* also obtained this relation and proved that $\tau_{\mathrm{D}}^{\mathrm{a}}$ equals τ_{D}. Substituting $T = T_{\mathrm{R}} + \mathrm{i}T_{\mathrm{I}}$ into equation (6) gives

$$\tau_{\mathrm{T}}^{\mathrm{a}} = (\hbar/|T|^2)(T_{\mathrm{R}}(\partial T_{\mathrm{R}}/\partial\Delta V_{\mathrm{I}}) + T_{\mathrm{I}}(\partial T_{\mathrm{I}}/\partial\Delta V_{\mathrm{I}}))|_{\Delta V_{\mathrm{I}}=0} \tag{8}$$

and the local Larmor transmission time is given by [10]

$$\tau_{z\mathrm{T}} = -\hbar\,\partial\varphi_{\mathrm{T}}/\partial\Delta V_{\mathrm{R}} = (\hbar/|T|^2)(-T_{\mathrm{R}}(\partial T_{\mathrm{I}}/\partial\Delta V_{\mathrm{R}}) + T_{\mathrm{I}}(\partial T_{\mathrm{R}}/\partial\Delta V_{\mathrm{R}}))|_{\Delta V_{\mathrm{R}}=0} \tag{9}$$

where the phase shift $\varphi_{\mathrm{T}} = \tan^{-1}(T_{\mathrm{I}}/T_{\mathrm{R}})$. If the transmission probability amplitude T is a complex analytic function of the complex potential $\Delta V = \Delta V_R + i\,\Delta V_I$, the Cauchy–Riemann condition yields

$$\partial T_R/\partial \Delta V_R = \partial T_I/\partial \Delta V_I \qquad \partial T_R/\partial\, \Delta V_I = -\partial T_I/\partial \Delta V_R. \tag{10}$$

From equations (8)–(10), we find that

$$\tau_T^a = \tau_{zT}. \tag{11}$$

We have $\tau_R^a = \tau_{zR}$ under the same conditions.

It should be noted that the potential variation ΔV is confined in the region $0 \leqslant z \leqslant d$ in the above definitions, i.e.

$$V = V_0(z) + \Delta V\,\theta(z)\theta(z-d). \tag{12}$$

Now we assume that ΔV is not confined in the barrier, i.e. $V = V_0(z) + \Delta V$ for the whole space. In this case, it is easy to prove that $\partial\varphi_T/\partial E = -\partial\varphi_T/\partial\Delta V_R$ from (3), and the transmission phase time [4, 5] $\tau_T^\varphi = \hbar\,\partial\varphi_T/\partial E$ equals the local Larmor time τ_{zT}. Hence, for the imaginary potential and the magnetic field covering the whole space, we have

$$\tau_T^a = \tau_{zT} = \tau_T \tag{13}$$

if T is a complex analytic function of ΔV. The same results can be obtained for the reflection time in the special case.

Finally, we calculate the tunnelling times for a double-barrier structure:

$$V(z) = \begin{cases} V_0 & 0 < z < b_1, \quad a + b_1 < z < a + b_1 + b_2 = d \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

with $V_0 = 0.3$ eV and the electron effective masses equal to $0.1m_0$ and $0.067m_0$ at the barriers and in the other regions, respectively, for the GaAs/GaAlAs system.

Matching the wavefunction and its first derivative divided by the effective mass at the interface of adjacent layers gives a system of homogeneous equations. The transmission and reflection probability amplitudes T and R can be obtained from the equations. The rough approximation of taking $(T(V + \Delta V_I) - T(V))/\Delta V_I$ to be $\partial T/\partial \Delta V_I$ with $\Delta V_I = -10^{-5}$ eV confined in the barrier $0 < z < d$ is made in the numerical calculation. We plot τ_T^a and τ_T^φ in figure 1(a) and τ_R^a and τ_R^φ in figure 1(b) versus incident electron energy E for a double-barrier structure with $b_1 = 30$ Å, $b_2 = 20$ Å, and $a = 50$ Å. As electrons impinge on the barrier from the right-hand side ($z = d$), the transmission times τ_T^a and τ_T^φ are still as described by figure 1(a), and the reflection times are plotted in figure 2. The results show that τ_T^a and τ_R^a coincide with τ_T^φ and τ_R^φ, respectively, except as E approaches zero or when the transmission probability is too small. As E approaches zero, we have $\tau^\varphi \to \infty$ and $\tau^a \to 0$. Taking it that ΔV_I covers the whole space, the numerical results for τ_T^a and τ_R^a are equal to τ_T^φ and τ_R^φ, respectively, over the whole energy region. We think that the imaginary potential confined in the barrier is more reasonable than the one that is not confined in the gedanken experiments. A precise measurement of the incident flux is difficult with an imaginary potential that covers the whole space.

It can be found that reflection times take negative values as the incident electron energy approaches some resonant position where the reflection probability takes a minimum value. The negative value of τ_R^a was used to reject the separation of τ_T^a and τ_R^a [7]. However, a negative reflection phase time also has physical meaning [2].

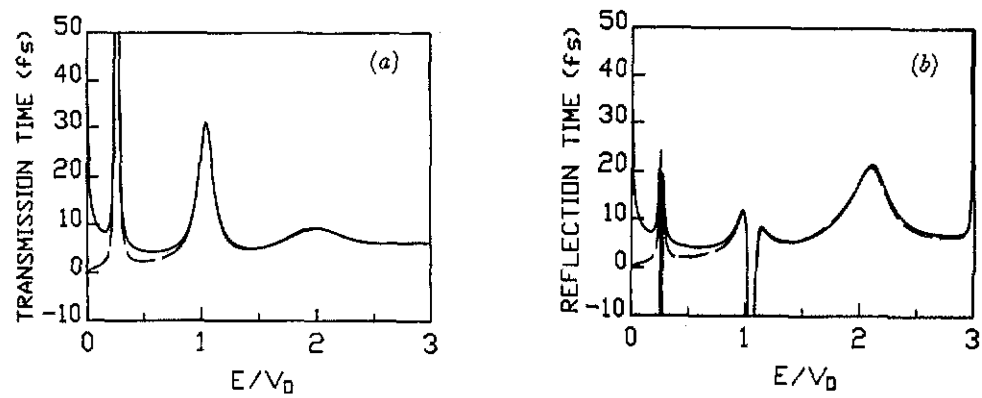

Figure 1. The tunnelling times versus incident electron energy E for a double-barrier structure with $b_1 = 30$ Å, $b_2 = 20$ Å, $a = 50$ Å, and $V_0 = 0.3$ eV: (*a*) transmission times; (*b*) reflection times; τ_R^a and τ_T^a (dashed line) and τ_R^φ and τ_T^φ (solid line).

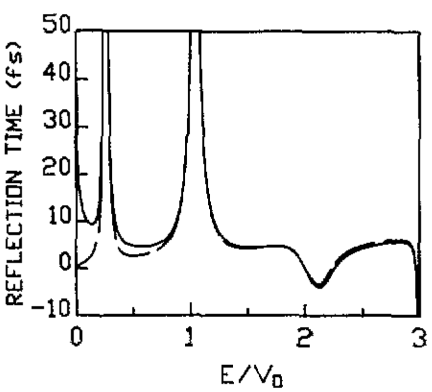

Figure 2. The reflection times versus incident electron energy E for a double-barrier structure with $b_1 = 21$ Å, $b_2 = 30$ Å, $a = 50$ Å, and $V_0 = 0.3$ eV; τ_R^a (dashed line) and τ_R^φ (solid line).

It is possible that the extrapolated 'peak' or the gravity of the reflected wavepacket emerges before the peak of the incident wavepacket impinges on the barrier. Applying equations (6) and (7) to calculate the mean time spent by *transmitted or reflected* electrons in a region on the far side of a barrier does not give correct results [7], and the same question also exists for the local Larmor times [1, 10]. This result may indicate that τ_T^a and τ_R^a cannot be defined locally. Finally, τ_T^a and τ_R^a are better candidates than the phase times in the non-local case. It should be noted that the phase times cannot be defined locally.

We have shown that the dwell time, the local Larmor times, and the phase times can be constructed by inducing a recombination process in the barrier region or the whole space. We believe that the tunnelling times derived from the absorption probabilities are the best of the various candidates.

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